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By Silvester van Koten and
Andreas Ortmann

Working paper series : REFGOV-IFM-14

Working Paper Series 313
(ISSN 1211-3298)

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CERGE-EI
Prague, December 2006

European FP6 Integrated Project – Coordinator: Centre for Philosophy of Law , UCLouvain – http://refgov.cpdr.ucl.ac.be
WP –IFM-14

ISBN 80-7343-109-2 (Univerzita Karlova. Centrum pro ekonomický výzkum a doktorské studium)
ISBN 80-7344-098-9 (Akademie věd České republiky. Národohospodářský ústav)

Bidding Behavior when One Bidder and the Auctioneer Are Vertically Integrated

Implications for the Partial Deregulation of EU Electricity Markets*

Silvester van Koten†

CERGE-EI‡

Abstract

When a bidder (referred to as the privileged bidder) is residual claimant to a part of the revenue from an auction with two bidders whose valuations are independently and identically distributed, bidding incentives are changed. Specifically, the privileged bidder will bid more aggressively to increase the auction revenue. Indeed, the privileged bidder is more likely to win the auction and the good is sold for a higher price. However, since the auction is now inefficient, welfare is decreased.

These results are of interest for regulators of the EU electricity industry. The extant EU regulatory framework allows for profits from new cross-border transmission lines (so-called interconnectors) to be unregulated and for incumbent Vertically Integrated Utilities (VIUs) to have ownership of generating and transmission activities. When electricity generators have to secure transmission rights in an auction, the VIU, because of its combined ownership of generation and transmission activities, is in the position of a privileged bidder. The VIU will secure a higher profit, while competing electricity generators will earn less because they are less likely to gain transmission rights and, in any case, pay a higher price for it.

Keywords: asymmetric auctions, bidding behavior, electricity markets, regulation, vertical integration.

JEL classification code: L43, L51, L94, L98

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* I want to thank Libor Dušek, Dirk Engelmann, Peter Katuščák, Avner Shaked and Sergey Slobodyan for their helpful comments. Special thanks to Andreas Ortmann. I thank the REFGOV Integrated project funded by the 6th European research framework programme, CIT3-513420, for financial support..

† Email: Silvester.VanKoten@cerge-ei.cz, slvstr@gmail.com.

‡ A joint workplace of the Center for Economic Research and Graduate Education, Charles University, Prague, and the Economics Institute of the Academy of Sciences of the Czech Republic.
Address: CERGE-EI, P.O. Box 882, Politických vězňů 7, Prague 1, 111 21, Czech Republic.

Abstrakt

V situaci, kdy má jeden z dražitelů (tzv. privilegovaný dražitel) reziduální nárok na část výnosu z aukce, které se účastní spolu s dalším dražitelem, a kdy jsou ocenění obou dražitelů navzájem nezávisle a identicky distribuována, jsou jeho poptávkové motivy významně pozměněny. Privilegovaný dražitel bude nabízet agresivněji s cílem zvýšit výnos z aukce. Je více pravděpodobné, že privilegovaný dražitel vyhraje aukci a zboží je prodáno za vyšší cenu. Vzhledem k tomu, že aukce je v takovém případě neefektivní, celkový blahobyt je snížen.

Tyto závěry jsou zajímavé pro regulátory elektroenergetiky v EU. Současná regulace v EU umožňuje, aby zisky z nových mezistátních přenosových vedení (tzv. spojovacích vedení) byly neregulované, a aby stávající Vertikálně Integrované Firmy (VIF) mohly vlastnit jak proces vyrábění tak i přenosu (transmise). Jestliže si výrobci elektřiny musí zabezpečit svá práva transmise v aukci, VIF jsou v pozici privilegovaného dražitele vzhledem ke kombinovanému vlastnictví výroby a transmise. VIF si tak zajistí vyšší zisky, zatímco konkurenční výrobci elektřiny vydělají méně, protože mají menší šanci získat práva na transmisi a pokud je získají tak platí vyšší cenu.

1. Introduction

I examine bidding behavior and outcomes of first-price and second-price auctions when one bidder, the privileged bidder, owns a part γ of the auctioneer. In essence, the privileged bidder receives a kickback: when he wins, he receives a refund of γ times his bid; when he looses, he receives γ times the bid of the other bidder. His ownership in the auctioneer affects the optimal bidding of the privileged bidder. Below, I call this the Any- Bid-Kickback scheme.

The present paper is motivated by the current process of liberalization of the electricity market of the European Union (EU). The two main activities of the electricity industry are generation (the production of electricity), which is done by electricity generators, and transmission (the transport of electricity over long distances), which is done by a Transmission System Operator (TSO). While electricity generation has become more competitive, transmission activities have kept characteristics of a natural monopoly (Pittman, 2003), necessitating continued regulation.

Moreover, the ownership of both generation and transmission by one company, from here on referred to as cross-ownership, impedes free competition in generation. The cross-ownership of generation and transmission by one company is the remainder of the dominance of vertically integrated state monopolies in the industry (Verbong et.al., 2002). These vertically integrated monopolies are also referred to as Vertically Integrated Utilities (VIU). Even though an active policy of liberalization has been pursued in the EU, VIUs usually still own not only many of the generation facilities, but also all, or almost all, of the transmission infrastructure. To facilitate competition, new generator entrants must be allowed to use the transmission infrastructure of the VIU. This brings about a conflict of interests, as the VIU would like to curb competition by allocating minimal infrastructure capacity to competing new entrants.

Regulatory policy is currently guided by DIRECTIVE 2003/54/EC [1] and REGULATION 1228/2003.[2] I analyze whether these laws address effectively the problems caused by the cross-ownership of generation and transmission by VIUs. The analysis is focused on cross-border transmission lines, also referred to as interconnectors. I discern three basic regulatory policy elements that are relevant in this respect:

1. The method of transmission capacity allocation
2. The unbundling policy
3. The regulation of transmission income

I now discuss these regulatory policy elements in this order.

1. The method of transmission capacity allocation

The first regulatory policy element is the method of transmission capacity allocation. The EU has defined the principles of transmission capacity allocation in DIRECTIVE 2003/54/EC ("principle 6"):

> "For competition to function, network access must be *non-discriminatory*, *transparent* and *fairly priced*." (my italics)

Along the same lines, REGULATION 1228/2003 (article 6.1), defines the principles of cross-border transmission capacity allocation as follows:

> "Network congestion problems shall be addressed with *non-discriminatory market based* solutions which give *efficient economic signals* to the market participants and transmission system operators involved" (my italics)

To derive a recommendation on the optimal design of a cross-border management regime, the European Commission commissioned a consultancy company.

[1] Directive 2003/54/EC of 26 June 2003 of the European Parliament and of the Council concerning common rules for the internal market in electricity and repealing Directive 96/92/EC (OJ 2003 L 176/37)
[2] Regulation (EEC) No 1228/2003 of the European Parliament and of the Council on Conditions for Access to the Network for Cross-Border Exchanges in Electricity (OJ 2003 L 176/1).

CONSENTEC (2004) derived from DIRECTIVE 2003/54/EC and REGULATION 1228/2003 three criteria (non-discriminatory, market-based, and efficient) for cross-border transmission capacity allocation and recommended as the appropriate mechanism (explicit or implicit) auctions. This recommendation happened to coincide with the already widely used practice of allocations by explicit auctions; in 2004 the interconnector transmission capacity between countries was allocated by explicit auctions on 14 border crossings[3] (European Transmission System Operators, 2004). Indeed, the European Transmission System Operators (ETSO) also favor the implementation of auctions on interconnection (ETSO, 2006)

2. The unbundling policy

The second regulatory policy element is the implementation of an unbundling policy. To assure the impartial handling of allocating transmission capacity, DIRECTIVE 2003/54/EC ("principle 8") requires the VIU to move transmission activities into a legally independent subsidiary;

> "In order to ensure efficient and non-discriminatory network access it is appropriate that the distribution and transmission systems are operated through *legally separate* entities where vertically integrated undertakings exist." (my italics)

In this so-called legal unbundling, the VIU prima facie loses its influence on the day-to-day operational decisions of the TSO. However, as long as the VIU keeps formal ownership of the transmission system, the VIU remains a residual claimant in the auction activities. In essence, the present manuscript analyzes the potential ramifications of the state of affairs.

As of the writing of this article, in 14 out of the 25 EU member countries[4] and in all 4 official candidate countries,[5] a VIU owns the transmission activity. In 8 of these 14 EU member countries not even effective legal unbundling has been implemented; the TSO does not have an independent board of directors (Commission of the European

[3] This involves countries such as The Netherlands, Belgium, Germany, France, Austria and Italy.
[4] This includes Austria, Belgium, Czech Republic, France, Germany and Ireland.
[5] The official candidate countries are Bulgaria, Croatia, Romania and Turkey.

Communities, 2005). An example where one bidder and the auctioneer are vertically integrated is the market configuration around the transmission line (called an interconnector) between The Netherlands and Belgium.[6]

3. The regulation of transmission income

The third regulatory policy element is the regulation of transmission income. Currently, almost all existing transmission networks in the world are being regulated. Regulation, however, poses its own problems for economic efficiency (e.g., Averch and Johnson 1962; Joskow and Tirole 2003). A concern of particular interest in the current context is the investment in new transmission capacity. Brunekreeft, Neuhoff and Newbery (2006) argue persuasively that legally unbundled VIUs have little incentive to invest in transmission capacity. After all, more transmission capacity increases competition in the market for electricity generation and lowers the profits from generation activities[7] for VIUs. Also, when the profitability of the transmission investment is uncertain, the VIU faces the regulatory risk of having profitable lines taxed but those incurring losses not being subsidized, a point also made by Brunekreeft et al. (2006).

Already, the EU faces – quite likely as a consequence of current regulatory practices and the historic precedent that the transmission network was not intended to facilitate international power trade (CONSENTEC, 2004) – severe shortages of transmission capacity between countries. At present there is little prospect of the shortages being addressed any time soon (see e.g. Brunekreeft et al, 2006).

Allowing unregulated for-profit building of transmission lines, also referred to as merchant transmission investment, could reinstate the incentives to invest in new transmission capacity. Indeed, the EU has created a legal base for unregulated for-profit

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6 As of the writing of this article, SUEZ, a large international energy company, which can be thought of as a VIU, almost fully (96%) owns the largest electricity generator in Belgium, ELECTRABEL, and owns, directly and indirectly, over 50% of the Belgian TSO, ELIA (*http://finance.google.com/finance?cid=699341*). In this sense, SUEZ is a participant in the auction through ELECTRABEL, as well as a beneficiary of the auction for transmission capacity through ELIA.

7 See also Leautier (2001).

building of transmission lines through REGULATION 1228/2003, article 7 and the NOTE[8] that interprets DIRECTIVE 2003/54/EC and REGULATION 1228/2003 with respect to the third party access regime (further referred to as "THE NOTE"). It is believed that weakening the regulatory regime will make investment in interconnector capacity more attractive, thereby eventually alleviating the shortage of capacity (Brunekreeft et. al., 2006). The US experience, both with shortages of transmission capacity and attempts to solve the problem through merchant transmission investments, suggest the viability of this strategy (Brunekreeft et. al., 2006).

A possible regulatory regime is suggested in THE NOTE:

> "Finally, for electricity, the infrastructure in question might merely be exempt from Article 6(6) of the regulation that deals with the use of congestion management revenues. Under such an exemption the infrastructure in question would be obliged to comply with the *congestion management guidelines* agreed under the Regulation. There would therefore be approval of the methodology by the regulator and Article 23(2) would apply. However, the developer would not be obliged to use the *revenues from congestion management methodologies* for the purposes set out in Article 6(6) of the Regulation. The regulator's right to intervene ex-post, as set out in 23(4), would therefore also be constrained." (my italics)

This regulatory regime, by not applying the third regulatory policy element of regulation of transmission income, allows the transmission owner to keep the profits of a line while still mandating a non-discriminatory, market-based and efficient method of allocating transmission capacity (the first regulatory policy element mentioned above).

While at present this particular regulatory regime has not been implemented anywhere in the EU yet, the fact that it is explicitly mentioned in THE NOTE suggests the

[8] NOTE OF DG ENERGY & TRANSPORT ON DIRECTIVES 2003/54-55 AND REGULATION 1228\03 IN THE ELECTRICITY; EXEMPTIONS FROM CERTAIN PROVISIONS OF THE THIRD PARTY ACCESS REGIME, 30.1.2004.

possibility that it will be used in the near future to deal with the regulatory problems discussed.

Here I analyze whether auctions – as the most likely instantiation of such a particular regulatory regime -- retain their favorable features (non-discriminatory, market-based and efficient) in such a setting. I especially focus on the role of incomplete (legal) unbundling as opposed to complete (ownership) unbundling of the transmission ownership (the second regulatory policy element mentioned above).

The remainder of this paper is organized as follows. In the next section I will analyze the effects of Any-Bid-Kickback schemes. I first sketch the general setup and then approximate numerically the equilibrium bidding functions of the bidders in a first price auction and show the effects on profits and welfare. I then determine the equilibrium bidding functions of the bidders in a second price auction and show the effects on profits and welfare. Having obtained these results, I relate my findings to the existing literature. I conclude by discussing the implications of my results for the EU policy of allowing incomplete (legal) unbundling of the VIUs in the EU electricity market.

2 Analysis of auctions with any-bid-kickback schemes

I consider the case where the transmission owner is unregulated, the transmission capacity is auctioned off to the highest bidder, and the transmission facility is legally independent but (partly) owned by a generator. The generator is therefore residual claimant of (a part of) the auction revenue. The marginal costs of generators are assumed to be unknown to competitors.[9] I analyze the outcomes of first price auctions and second price auctions and discuss the welfare consequences of vertical integration in the electricity industries for various degrees of ownership.

More specifically, I will study Any-Bid-Kickback (ABK) schemes in a set-up with two bidders. One is a privileged bidder who owns part γ of the auctioneer. The other bidder does not own any part of the auctioneer and below is called the independent bidder.

[9] This is a natural assumption; unregulated, competitive generators have neither the obligation nor the incentive to share this commercially sensitive information (Leautier, 2001).

In the energy industry, the good to be auctioned off is the right to use the transmission line to sell electricity in a relatively distant location. The value of the good is therefore the profit that could be made by selling electricity in this distant location. This profit is equal to the difference between the price in the distant location and the costs of the generator, because the transmission capacity is fixed and small relative to the total demand (see, e.g., CONSENTEC, 2004). In line with the empirical evidence, I assume that the bidders cannot influence the final price in the distant location.

2.1 Setup

I analyze first price and second price auctions with two risk-neutral bidders. The privileged bidder Y owns a part γ of the auctioneer and is therefore under all circumstances a benefactor of the auction revenue. Below I refer to this configuration as cross-ownership. The other bidder is the independent bidder X. I assume that the auctioneer auctions off the transmission capacity as one indivisible good.[10]

Assumption

The value of the good to a bidder is drawn from a uniform distribution $v_i \in [0,1]$ where $i \in \{X, Y\}$. Values are private and independent.[11]

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[10] While transmission capacity is usually auctioned in many units of 1 GW, I restrict my focus to single-unit auctions. Excluding multi-unit auctions simplifies the analysis of ABK-schemes in auctions. Multi-unit auctions mostly do not have efficient outcomes and mostly cannot be analytically solved, which complicates the task of demonstrating the effects of ABK-schemes.

[11] The above assumption is motivated by the fact that there exist price differences between countries that can be profitably exploited. The size of the profit is dependent on the costs of generating electricity. As a generator does not know the cost of his competitors, he treats it as a random variable, drawn from a distribution that for sake of simplicity I will assume to be uniform. The costs, $\tilde{c}_Y, \tilde{c}_X \in [0,1]$, are private and independent. The random costs drive the dynamics of the bidding behavior. In electricity generation, there is also a common cost component, mainly gas or oil prices. I assume that the size of these common cost components are common knowledge and that they are identical for both generators. As a result, these common cost components are inconsequential for the bidding behavior as long as the common cost component is $C \in [0, P-1]$ where P is the price in the distant location. Let the common profit be given by $R = P - C - 1$, then the random value is $v_i = 1 - \tilde{c}_i$, and the total value to a bidder is $R + v_i = P - C - \tilde{c}_i$.

I assume that both bidders have identical value distributions. At the outset, the bidders are therefore symmetrical. Given his value realization, the privileged bidder Y chooses his optimal bid, b_Y. In line with the literature, I assume that there exists a differentiable, strictly increasing bidding strategy $b_Y[\cdot]$ that maps the privileged bidder's realized value $v_Y \in [0,1]$ into his bid $b_Y[v_Y]$.[12] The bidding strategy $b_Y[\cdot]$ has an inverse $y[\cdot]$ such that $y[b_Y[v]] = v$. Analogously, the optimal bid of the independent bidder X, b_X, is determined by her bidding strategy $b_X[\cdot]$ that maps her realized value $v_X \in [0,1]$ into her bid $b_X[v_X]$. The strategy $b_X[\cdot]$ has an inverse $x[\cdot]$, such that $x[b_X[v]] = v$.

Given this setup, the privileged bidder Y, when he bids b, has a probability of winning equal to $x[b]$; in the same way the independent bidder X , when she bids b, has a probability of winning equal to $y[b]$.[13]

2.2 The second price auction

I first analyze the effect of the privileged bidder Y owning a share of the auctioneer in second price auctions.[14] The bidding functions of X and Y in second price auctions have been found earlier by Burkart (1995) and Ettinger (2002b).[15]

$$b_X = v_X$$

$$b_Y = \frac{\gamma + v_Y}{\gamma + 1}$$

Figure 1

[12] The strategies $b_Y[\cdot]$ and $b_X[\cdot]$ (and their respective inverses $x[\cdot]$ and $y[\cdot]$) are dependent on the ownership share γ. For notational convenience I will not include the variable "γ" in the derivation to follow.

[13] See, for example, Krishna (2002). For easy reference, a proof may be found in proposition 1 in the appendix.

[14] In the second price auction the highest bidder wins and pays the bid of the second highest bid.

[15] For the more general case when the privileged bidder faces n independent competing bidders, see proposition 2 in the appendix.

Figure 1 illustrates the bidding by the privileged bidder and the independent bidder. Because of his ownership holding in the auctioneer, the privileged bidder Y bids more aggressively. The more aggressive bidding has several interesting effects:

a. Y is more likely to win the auction.
b. Y earns lower profits from his generation activities.
c. The revenue of the auctioneer (or revenue from the auction) is higher.
d. Y earns a higher compound profit.
e. X earns lesser profits.
f. Welfare decreases.
g. The strategic profit[16] of Y increases.

The intuition for these effects is as follows;[17]

Ad a. Y is more likely to win the auction because Y now bids more aggressively than X.

Ad b. Y earns lower profits from his generation activities because Y takes his part of the auctioneer revenue in account. Y thus chooses a different bidding function while the original bidding function is still available. Therefore, his new bidding function cannot be optimal. Indeed, the new bidding schedule of Y maximizes the compound profit (the profit including his share of the auction revenue) rather than the profits from generation activities only. As a result, the generation profits are lower now.

[16] The ownership share γ has a direct and a strategic effect on the compound profit. The direct effect translates into what I will refer to as the "passive" profit and is due to the fact that Y receives proportion γ of the auction revenue. The "passive" profit is the profit that Y would receive when he owns the proportion γ, but bids as if his ownership share was zero. The strategic effect translates into what I will refer to as the "strategic" profit and is due to Y changing his bidding schedule. The strategic profit can be found by subtracting the passive profit from the compound profit.
[17] The effects are all described by *ex-ante expected* measures (before bidding and before concrete values have been realized). For the exact quantitative effects, see the explicit formula in proposition 3 in the appendix.

Ad c. The revenue of the auctioneer (or revenue from the auction) is higher. When Y loses, the losing bid of Y is higher, and hence X pays more for the good. When Y wins, Y either pays the same (Y would have won with or without the ownership share) or Y pays more (Y would have lost without the ownership share).

Ad d. Y earns a higher compound profit because Y receives a larger part of the auction revenue; hence Y would earn a strictly higher profit even if Y did not change his bidding function.

Ad e. X earns lesser profits because X is less likely to win the auction and when X wins, she pays a higher price.

Ad f. Welfare decreases because the auction is now inefficient. While Y has the same value distribution as X, Y now wins in some cases when he does not have the highest value for the good, because Y now bids more aggressively than X.

Ad g. The strategic profit of Y increases. Y changes his bid, while the old one is still available. As there is a unique optimal bid, this argument reveals that Y must be better off with his new bid.

As the bids of Y are strictly more aggressive with increasing ownership share γ, all effects increase strictly in the ownership share γ. As is illustrated in Figure 2, the welfare loss can be as high as 6.6%, while the strategic profit can be up to 16.7%.

Figure 2

2.3 The first price auction

In this section, I will analyze the effect of the privileged bidder Y owning a share of the auctioneer in first price auctions.[18] When the privileged bidder Y owns a share γ of the auctioneer and has value realization v_Y while bidding b_Y, his expected compound profit is[19]

$$1)\quad \pi^{Y}_{Compound}\left[b_Y\right] = \int_0^{x[b_Y]} v_Y - (1-\gamma)b_Y dv_X + \int_{x[b_Y]}^{1} \gamma b_X dv_X$$

$$= x\left[b_Y\right]\left(v_Y - (1-\gamma)b_Y\right) + \gamma\left(\overline{b} - b_Y x[b_Y] - \int_{b_Y}^{\overline{b}} x[\beta]d\beta\right),$$

where $\overline{b}$ is the maximum bid.

The expected profit of the independent bidder X with value realization v_X and bidding b_X is

$$2)\quad \pi^{X}\left[b_X\right] = \int_0^{y[b_X]} v_X - b_X[v_X]dv_Y$$

$$= y[b_X]\left(v_X - b_X\right).$$

To calculate the maximal profit for Y, differentiate equation 1) with respect to b_Y, set it equal to zero and substitute $y[b]$ for v_Y:

$$3)\quad (y[b]-b)x'[b] = (1-\gamma)x[b].$$

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[18] In a first price auction the highest bidder wins and pays his own bid.

[19] See, for example, Krishna (2002). For easy reference, the proof may be found in proposition 4 in the appendix. Note that the outcome does not depend on the common profit factor R (as defined in footnote 11.). It's a Nash equilibrium, for any value realization, for a bidder to bid at least R. Denote the optimal bid as $R+\breve{b}_Y$, where $\breve{b}_Y$ is the amount that is bid in addition to R. Then the expected compound profit of the privileged bidder Y becomes:

$$\pi^{Y}_{Compound}\left[\breve{b}_Y\right] = \int_0^{x[\breve{b}_Y]} R + v_Y - (1-\gamma)\left(R+\breve{b}_Y\right)dv_X + \int_{x[\breve{b}_Y]}^{1} \gamma\left(R+\breve{b}_X\right)dv_X$$

$$= x[\breve{b}_Y]\gamma R + \int_0^{x[\breve{b}_Y]} v_Y - (1-\gamma)\left(\breve{b}_Y\right)dv_X + (1-x[\breve{b}_Y])\gamma R + \int_{x[\breve{b}_Y]}^{1} \gamma \breve{b}_X dv_X$$

$$= \gamma R + \int_0^{x[\breve{b}_Y]} v_Y - (1-\gamma)\left(\breve{b}_Y\right)dv_X + \int_{x[\breve{b}_Y]}^{1} \gamma \breve{b}_X dv_X$$

which is, apart from a shift factor equal to γR, identical to the derivation without a common profit factor R.

To calculate the maximal profit for X, differentiate equation 2) with respect to b_X, set it equal to zero and substitute $x[b]$ for v_X:

4) $(x[b]-b)\cdot y'[b]=y[b]$.

Equations 3) and 4) form a system of differential equations that can be explicitly solved for $x[b]$ and $y[b]$ when either $\gamma=0$ or $\gamma=1$.[20]

As long as the ownership share γ is equal to zero, $\gamma=0$, bidders X and Y are symmetrical and choose bidding strategy $b[v]=\frac{1}{v}\int_0^v xdx=\frac{1}{2}v$ (see e.g. Krishna, 2002).[21] In the case of full ownership, $\gamma=1$, bidder X chooses the same bidding strategy, $b_X=\frac{1}{2}v_X$, while bidder Y chooses a more aggressive bidding strategy, $b_Y=v_Y$.[22] For $0<\gamma<1$, I could not find analytical expressions for the bidding functions. Figure 3 therefore shows numerical approximations[23] of the bidding functions for $0<\gamma<1$ together with the bidding function found above for $\gamma=1$.[24]

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[20] For the general case with one privileged bidder and n independent risk-neutral bidders, see proposition 5 in the appendix.

[21] More generally, for any symmetrical differentiable cumulative distribution of values $F[\cdot]$, the auction has the solution $b[v]=\frac{1}{F[v]}\int_0^v xf[x]dv$ (see e.g. Krishna, 2002).

[22] Intuitively, when $\gamma=1$, Y receives the full amount of any bid paid. Therefore Y does not have to take bidding costs into account and has a lower bound on the expected profit of $\min[v_Y,b_X]$. Now an argument similar to that for truthful bidding in second-price auctions applies. Suppose Y has value v_Y. If Y makes a bid lower than his value $b_Y<v_Y$, then with a positive probability X wins with a bid, b_X,which higher than the bid of Y but lower than the value of Y, $b_Y<b_X<v_Y$. In this case Y can guarantee himself a higher profit at no costs by bidding his value, $b_Y=v_Y$. A similar argument establishes that Y will not make a bid higher than his value. Hence, Y bids $b_Y=v_Y$ and has an expected profit of $\max[v_Y,b_X]$. The inverse bidding function of Y is then $y[b]=b$. Substituting $y[b]=b$ in equation 4 results in $x=2b$. Taking the inverse gives $b_X=\frac{1}{2}v_X$.

[23] Proposition 6 in the appendix lays out the necessary restrictions that the bidding strategies must fulfill.

[24] Note that there is a discontinuity at $\gamma=1$. If and only if $\gamma=1$, then bidding $b_Y=v_Y$ is a weakly dominant strategy for Y. Suppose $\gamma=1-\delta$ (for small $\delta>0$), then if X sticks with his strategy $b_X=\frac{1}{2}v_X$, then Y would never bid more than $\frac{1}{2}+\varepsilon$ (for small $\varepsilon>0$). At $v_Y=\frac{1}{2}+\varepsilon$ there would be

Figure 3: the bidding functions for X and Y.

Effects on profits and welfare

The bidding functions in Figure 2 demonstrate that an increased ownership share in the auctioneer results in the privileged bidder Y bidding more aggressively. The result, and the intuition, is similar to what we found in the second price auction; because of his ownership holding in the auctioneer, the privileged bidder Y bids more aggressively. The more aggressive bidding has several interesting effects:[25]

a. Y is more likely to win the auction. The probability of winning for Y increases from $\frac{1}{2}$ for no ownership to $\frac{3}{4}$ for full ownership.

b. Y earns lower profits from his generation activities.

c. The revenue of the auction is higher; it increases from $\frac{1}{3}$ for no ownership to $\frac{13}{24}$ for full ownership.[26]

d. Y earns a higher compound profit; it increases from $\frac{1}{2}$ for no ownership to $\frac{13}{24}$ for full ownership.

e. X earns lesser profits; they decrease from $\frac{1}{6}$ for no ownership to $\frac{1}{12}$ for full ownership.

f. Welfare decreases; it falls from $\frac{2}{3}$ for no ownership to $\frac{15}{24}$ for full ownership.

g. The strategic profit increases; it increases from 0 for no ownership to $\frac{1}{24}$ for full ownership.

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a mass point which in turn would create an incentive for X to try to overbid it whenever her value is larger ($v_X > \frac{1}{2} + \varepsilon$). Therefore, once $\gamma < 1$, bidding $b_Y = v_Y$ cannot be an equilibrium strategy for Y. For an equilibrium in pure strategies to exists at all, the bidding functions of X and Y must have the same bid for $v_Y = v_X = 1$. This is the case in the strategies shown in Figure 3; there are no mass points, and the density of Y's bids is continuous, excluding the possibility for X to improve her profits by deviating from her strategy. This implies that the maximum bid $\overline{b}$ converges to 1 when the ownership share γ goes to 1.

[25] Explicit calculations can be found in proposition 7 in the appendix.

[26] For the case of full ownership this is equal to the auction revenue in a second price auction.

3. Related literature

The asymmetric type of bidding behavior discussed in this manuscript also features prominently in the literature on “toehold” auctions (Bulow, Huang and Klemperer, 1999; Singh, 1998; Ettinger, 2002a) which focuses on take-over contests where one or both bidders have an ownership share (“toehold”) in the target company. As a result, the winning as well as the losing bidder receive a refund proportional to their respective toeholds on the price paid. It is well-established theoretically that the size of the toeholds and the (a)symmetry of the toeholds influence the aggressiveness of the bidding (Engelbrecht-Wiggans, 1994; Burkart 1995; Bulow et. al. 1999; Singh, 1998; Ettinger, 2002a).

Engelbrecht-Wiggans (1994) modeled first price and second price auctions with common values where both bidders have a symmetrical ownership share. Bulow et. al. (1999) modeled asymmetric ownership shares for first price and second price auctions with common values, where the ownership shares are strictly positive but asymmetric for both bidders. Bulow et. al. (1999) showed that in first price auctions the bidder with the larger ownership share bids more aggressively, while the bidder with the smaller ownership share bids *less* aggressively. In my model the bidders have private values and only the privileged bidder has a strictly positive ownership share.

Using the analytic and numerical solutions of the ABK scheme in the first price and second price auction, I determine the effect of the ownership share of the auctioneer on welfare and revenue. In my model this ownership share (the “toehold”) can be higher than 50% (up to 100%), something that would not make sense in the context of take-over contests, as considered by Burkart (1995), Bulow et. al. (1999) and Engelbrecht-Wiggans (1994).

4. Conclusion

My analysis suggests that under two prominent auction formats, an incompletely unbundled VIU will drive up the price of merchant cross-border transmission lines by aggressive bidding. Consequently, the amount of transmission allocated to the VIU-owned generator increases at the expense of independent competing generators. This increases the profits of the VIU, while causing welfare losses.

This result should be of interest to regulators of the EU electricity industry, as they might consider addressing the issue of underinvestment in capacity by allowing unregulated for-profit building of transmission lines. In such a setting, a VIU might be tempted – and in fact is likely, as I have shown above – not to allocate transmission capacity in a non-discriminatory and efficient manner. Most notably auctions - as the most likely instantiation of such a particular regulatory regime - lose their favorable features (non-discriminatory, market-based and efficient).
As a result, the competitive effect of new connection lines in the merchant model is smaller under legal unbundling than under ownership unbundling. This questions the claim of Brunekreeft et al (2006) that ownership restrictions are not much of a concern, as they assume that the owner will want to keep competitive pressure between the generators. My model shows that this is not the case as long as only legal unbundling is applied.

An obvious extension that I did not address in this paper is how this model translates to the setting of a multi-unit auction with more competing independent bidders. The transmission capacity is typically not auctioned as one indivisible good as I assumed in this paper, but in units of 1 GW. In this paper I point out the basic mechanism of a kickback scheme and its effect on profits and welfare. It seems likely that similar effects operate in multi-unit auctions. Additional questions that should be answered in a multi-unit auction framework are what are the effects of different payment rules (uniform price, discriminatory or Vickery auction) on incentives, profits and welfare. However, it can be complicated, if not impossible to solve explicitly for kick-back schemes in multi-unit auctions.

Another question of importance is how the analysis is modified when the privileged bidder does not directly own the auctioneer, but a holding company owns both the privileged bidder and the auctioneer.[27] When the holding company has imperfect

[27] This is a more realistic set-up for the example of the Belgium VIU Electrabel and the TSO Elia. The common ownership is now centralized in the large energy company SUEZ.

information about the value of transmission and the costs of the privileged bidder, then the holding company cannot determine the optimal action by the privileged bidder. The holding company, therefore, must delegate decisions to the privileged bidder. To make the privileged bidder maximize the holding company's profit, the privileged bidder must be offered a well-designed compensation scheme. Such a compensation scheme is known to leave the privileged bidder with an information rent. The existence of such delegated optimization could be expected to weaken the tendency of the privileged bidder to bid more aggressively. I take up this question in Van Koten (2006) and show that with a simple standard compensation scheme, the privileged bidder still has the tendency to bid more aggressively.

5. Appendix

For reference only: Proposition 1

When the privileged bidder Y bids b, the probability that his bid is the highest is $x[b]$. When the independent bidder X bids b, the probability that her bid is the highest is $y[b]$.

Proof

The privileged bidder Y with a realized value of v_Y will win when his bid $b_Y[v_Y]$ is larger than the bid of the independent bidder, $b_X[v_X]$;

$$b_X[v_X] < b_Y[v_Y] \Leftrightarrow$$

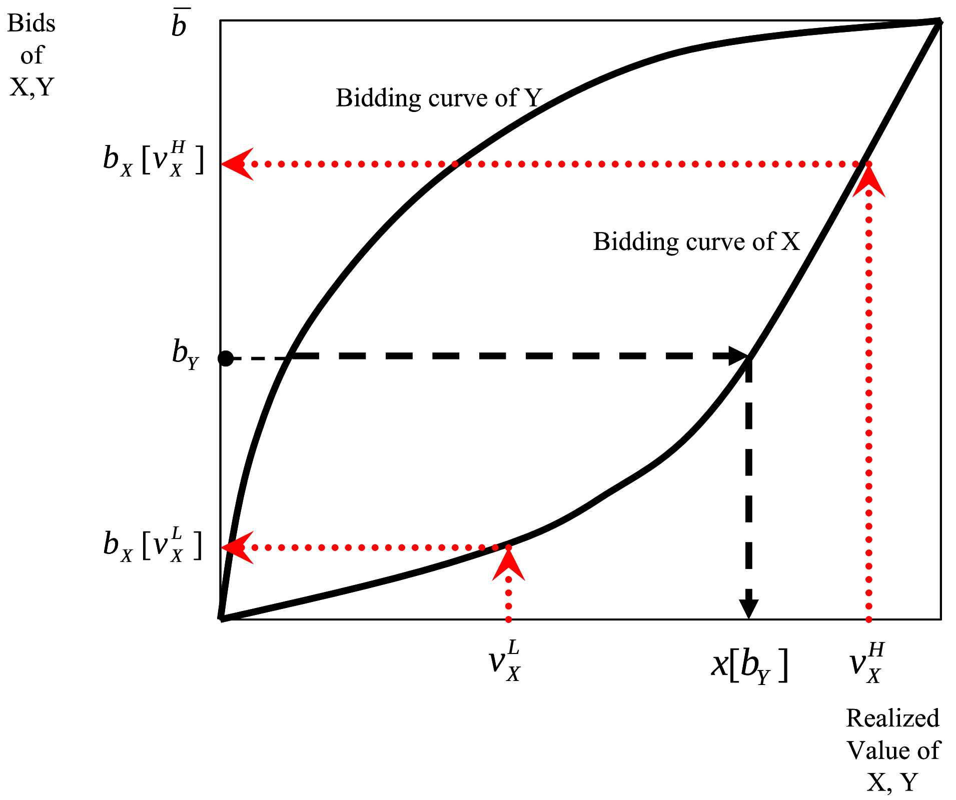
$$v_X < b_X^{-1}\left[b_Y[v_Y]\right] \equiv x\left[b_Y[v_Y]\right] \equiv x \circ b_Y[v_Y].$$

v_X represents the possible random values of bidder X, which are distributed with F. Therefore, $v_X < x \circ b_Y[v_Y]$ holds with probability $F[x \circ b_Y[v_Y]]$. I will solve for x as a function of b, so I will consider the probability of winning as $F[x[b]]$. Invoking the assumption that F is the uniform distribution on $\left[0,1\right]$, the probability of winning becomes $x[b]$. Likewise, the independent bidder X with a realized value of v_X wins with probability $y[b]$.

Graphical illustration

The derivation of the probability of winning is illustrated in the example in Figure 1. The horizontal axis gives the value realizations for a bidder, which ranges from zero to one by assumption. The vertical axis gives the bids of both X and Y, which range from zero to the maximum bid $\bar{b}$. A bidder makes the maximum bid when he has the maximum value, which is equal to 1 in this model. In the example, the bidding curve of Y stochastically dominates the bidding curve of X. Figure 4 shows a realization for which Y wins, $v_X^L < x[b_Y]$ and a realization for which Y looses $v_X^H > x[b_Y]$. Generally, Y wins when $v_X < x[b_Y]$. The probability of $v_X < x[b_Y]$ is $F\left[x[b_Y]\right]$, as v_X is distributed with cumulative distribution $F\left[\cdot\right]$. When $F\left[\cdot\right]$ is the cumulative uniform distribution on $\left[0,1\right]$, this probability is equal to $x[b]$.

Figure 4



In the above example the bidding curve of Y stochastically dominates the bidding curve of X, hence Y bids more aggressively than X. Therefore, when their value realizations are identically distributed, Y is more likely to win. We will see that when Y holds a positive ownership share γ in the auctioneer, then, as in this example, the bidding curve of Y will stochastically dominate the bidding curve of X.

Proposition 2

When in the second price auction the allied bidder faces n independent symmetric competing bidders, the bidding function for each of the n independent bidders X is $b_X = v_X$.

The bidding function for the allied bidder Y is implicitly determined by the equation

$$v_Y = b_Y(\frac{1}{n} + \gamma) - \frac{\gamma}{n b_Y^{\,n-1}}.$$

Proof:

In the second price auction, for X, it is a weakly dominant strategy to bid her true value. Therefore the bidding function of X is $b_X = v_X$

When the allied bidder Y faces n independent symmetric competing bidders, then his expected profit becomes

$$8') \quad \pi^{Y}_{Compound}\left[b_Y\right] = \int_0^{b_Y^n}\left(v_Y - (1-\gamma)b_X\left[v_X\right]\right)db_X^n + \gamma\int_{b_Y^n}^{1} b_Y\, db_X^n .$$

Differentiating this with respect to b_Y and setting equal to 0 gives

$$v_Y \cdot n b_Y^{\,n-1} - (1-\gamma)\left\{n b_Y^{\,2n-1}\right\} + \gamma\left\{1-(n+1)b_Y^n\right\} = 0\,;$$

$$v_Y \cdot n b_Y^{\,n-1} = b_Y^{\,n} - \gamma + \gamma n b_Y^n\,;$$

$$v_Y \cdot n b_Y^{\,n-1} = b_Y^{\,n}(1+\gamma n) - \gamma\,;$$

$$v_Y = b_Y(\frac{1}{n} + \gamma) - \frac{\gamma}{n b_Y^{\,n-1}}.$$

Proposition 3: Second price auction effects

All effects with * are ex-ante expected measure (before bidding and before concrete values have been realized).

a. The probability of winning* for the privileged bidder Y, $p^{Y\,wins}[\gamma] = \frac{\frac{1}{2}+\gamma}{1+\gamma}$, is strictly increasing in the ownership share γ.

Proof:

Using proposition 1, it follows that the privileged bidder Y with a realized value of v_Y wins with probability

$$p^{Y\,wins}[v_Y;\gamma] = x \circ b_Y[v_Y;\gamma] = \frac{\gamma + v_Y}{\gamma + 1}.$$

The expected proportion of auctions that is won by the privileged bidder Y is therefore

$$p^{Y\,wins}[\gamma] = \int_0^1 p^{Y\,wins}[v_Y;\gamma]dv_Y$$

$$= \int_0^1 \frac{\gamma + v_Y}{\gamma + 1} dv_Y$$

$$= \frac{\frac{1}{2} + \gamma}{1 + \gamma}.$$

b. The generation profit* of Y, $\pi^Y_{Generation}\left[\gamma\right] = \frac{1+2\gamma}{6\left(1+\gamma\right)^2}$, is strictly decreasing in the ownership share γ.

Proof:

$$\pi^Y_{Generator}\left[\gamma\right] = \int_0^1 P^{YWINS}\left(v_Y - E\left[b_X \mid b_X < b_Y\right]\right)dv_Y$$

$$= \int_0^1 \frac{\gamma + v_Y}{\gamma + 1}\left(v_Y - E\left[v_X \mid v_X < \frac{\gamma + v_Y}{\gamma + 1}\right]\right)dv_Y$$

$$= \int_0^1 \frac{\gamma + v_Y}{\gamma + 1}\left(v_Y - \frac{1}{2}\frac{\gamma + v_Y}{\gamma + 1}\right)dv_Y$$

$$= \left[\frac{\frac{1}{2}\gamma v_Y^2 + \frac{1}{3}v_Y^3}{\gamma + 1} - \frac{\gamma^2 v_Y + \frac{1}{3}v_Y^3 + \gamma v_Y^2}{2(\gamma + 1)^2}\right]_0^1$$

$$= \frac{1+2\gamma}{6\left(1+\gamma\right)^2},$$

and $\frac{d\pi^Y_{Generator}\left[\gamma\right]}{d\gamma} = \frac{-\gamma}{3(1+\gamma)^3} < 0$ for $\gamma > 0$.

c. The revenue* of the auctioneer, $m[\gamma]=\frac{1}{2}-\frac{1}{6(1+\gamma)^2}$, is strictly increasing in the ownership share γ.

Proof:

$$m[\gamma]=m^Y[\gamma]+m^X[\gamma]$$

$$=\int_0^1 P^{Y\,WINS}\cdot E\left[b_x \mid b_x<b_y\right]dv_y+\left(\int_{\frac{\gamma}{1+\gamma}}^1 P^{X\,WINS}\cdot E\left[b_Y \mid b_X>b_Y\right]dv_X\right)$$

$$=\int_0^1 \frac{\gamma+v_y}{\gamma+1}\cdot E\left[v_x \mid v_x<\frac{\gamma+v_y}{\gamma+1}\right]dv_y+\left(\int_{\frac{\gamma}{1+\gamma}}^1 \left((1+\gamma)v_X-\gamma\right)\cdot E\left[b_Y \mid v_X>\frac{\gamma+v_Y}{\gamma+1}\right]dv_X\right)$$

$$=\int_0^1 \frac{1}{2}\left(\frac{\gamma+v_y}{\gamma+1}\right)^2 dv_y+\left(\int_{\frac{\gamma}{1+\gamma}}^1 \left((1+\gamma)v_X-\gamma\right)\cdot E\left[\frac{\gamma+v_Y}{\gamma+1} \mid v_Y<(1+\gamma)v_X-\gamma\right]dv_X\right)$$

$$=\frac{1+3\gamma+3\gamma^2}{6(1+\gamma)^2}+\left(\int_{\frac{\gamma}{1+\gamma}}^1 \left((1+\gamma)v_X-\gamma\right)\cdot\frac{\gamma+\frac{1}{2}\left((1+\gamma)v_X-\gamma\right)}{1+\gamma}dv_X\right)$$

$$=\frac{1+3\gamma+3\gamma^2}{6(1+\gamma)^2}+\left[\frac{\frac{1}{3}(1+\gamma)^2 v_X^2-\gamma^2 v_X}{2(\gamma+1)}\right]_{\frac{\gamma}{1+\gamma}}^1$$

$$=\frac{1+3\gamma+3\gamma^2}{6(1+\gamma)^2}+\left(\frac{1+3\gamma}{6(1+\gamma)^2}\right)$$

$$=\frac{1}{2}-\frac{1}{6(1+\gamma)^2}.$$

d. The compound profit* of Y, $\pi^Y_{Compound}[\gamma]=\frac{1+3\gamma+3\gamma^2}{6(1+\gamma)}$, is strictly increasing in the ownership share γ.

Proof:

$$\pi^Y_{Compound}[\gamma]=\pi^Y[\gamma]+\gamma m[\gamma]$$

$$=\frac{1+2\gamma}{6(1+\gamma)^2}+\gamma\left(\frac{1}{2}-\frac{1}{6(1+\gamma)^2}\right)$$

$$=\frac{1+3\gamma+3\gamma^2}{6(1+\gamma)}.$$

Then $\frac{d\pi^{Y}_{Compound}[\gamma]}{d\gamma}=\frac{d\,\frac{1+3\gamma+3\gamma^2}{6(1+\gamma)}}{d\gamma}=\frac{2+6\gamma+3\gamma^2}{6(1+\gamma)^2}>0$.

e. The profit* of the independent bidder X, $\pi^{X}[\gamma]=\frac{3+8\gamma}{24(1+\gamma)^2}$, is strictly decreasing in the ownership share γ.

Proof:

$$m^{X}[\gamma]=\int_{\frac{\gamma}{1+\gamma}}^{1}P^{X\,WINS}\cdot\left(v_X-E\left[b_Y\mid b_X<b_Y\right]\right)dv_X$$

$$=\int_{\frac{\gamma}{1+\gamma}}^{1}\left((1+\gamma)v_X-\gamma\right)\cdot\left(v_X-E\left[b_Y\mid v_X>\frac{\gamma+v_Y}{\gamma+1}\right]\right)dv_X$$

$$=\int_{\frac{\gamma}{1+\gamma}}^{1}\left((1+\gamma)v_X-\gamma\right)\cdot\left(v_X-E\left[\frac{\gamma+v}{\gamma+1}\mid v_Y<(1+\gamma)v_X-\gamma\right]\right)dv_X$$

$$=\int_{\frac{\gamma}{1+\gamma}}^{1}\left((1+\gamma)v_X-\gamma\right)\cdot\left(v_X-\frac{\gamma+\frac{1}{2}\left((1+\gamma)v_X-\gamma\right)}{\gamma+1}\right)dv_X$$

$$=\int_{\frac{\gamma}{1+\gamma}}^{1}\left[\tfrac{1}{2}(1+\gamma)v_X^2-\gamma v_X+\frac{\gamma^2}{2(\gamma+1)}\right]dv_X$$

$$=\left[\tfrac{1}{6}(1+\gamma)v_X^3-\tfrac{1}{2}\gamma v_X^2+\frac{\gamma^2 v_X}{2(\gamma+1)}\right]_{\frac{\gamma}{1+\gamma}}^{1}$$

$$=\tfrac{1}{6}(1+\gamma)-\tfrac{1}{2}\gamma+\frac{\gamma^2}{2(\gamma+1)}-\tfrac{1}{6}(1+\gamma)\left(\frac{\gamma}{1+\gamma}\right)^3+\tfrac{1}{2}\gamma\left(\frac{\gamma}{1+\gamma}\right)^2-\frac{\gamma^2\left(\frac{\gamma}{1+\gamma}\right)}{2(\gamma+1)}$$

$$=\frac{(1-2\gamma)(\gamma^2+2\gamma+1)}{6(\gamma+1)^2}+3\frac{\gamma^3+\gamma^2}{6(\gamma+1)^2}-\frac{\gamma^3}{6(1+\gamma)^2}+\frac{3\gamma^3}{6(1+\gamma)^2}-\frac{3\gamma^3}{6(1+\gamma)^2}$$

$$=\frac{1}{6(\gamma+1)^2}.$$

f. The welfare*, $W[\gamma] = \frac{2}{3} - \frac{\gamma^2}{6(1+\gamma)^2}$, is strictly decreasing in the ownership share γ.

Proof:

Welfare is equal to:

$$W[\gamma] = \pi^Y[\gamma] + \pi^X[\gamma] + m[\gamma]$$

$$= \frac{1+2\gamma}{6(1+\gamma)^2} + \frac{1}{6(1+\gamma)^2} + \frac{2+6\gamma+3\gamma^2}{6(1+\gamma)^2}$$

$$= \frac{2}{3} - \frac{\gamma^2}{6(1+\gamma)^2} \text{ and } \frac{dW[\gamma]}{d\gamma} = -\frac{\gamma}{3(1+\gamma)^3} < 0.$$

Using $m[\gamma] = \frac{1}{2} - \frac{1}{6(1+\gamma)^2} = \frac{2+6\gamma+2\gamma^2}{6(1+\gamma)^2}$, $\pi^Y_{Generator}[\gamma] = \frac{1+2\gamma}{6(1+\gamma)^2}$ and

$\pi^X[\gamma] = \frac{1}{6(1+\gamma)^2}$.

g. The strategic profit* of Y, $\pi^Y_{Strategic}[\gamma] = \frac{\gamma^2}{6(1+\gamma)}$, is strictly increasing in the ownership share γ.

Proof:

Using the definition of the passive profit

$$\bar{\pi}^Y_{Passive}[\gamma] = \pi^Y_{Generator}[0] + \gamma m[0] = \frac{1}{6} + \gamma\frac{1}{3} = \frac{1+2\gamma}{6},$$

$$\pi^Y_{Strategic}[\gamma] = \pi^Y_{Compound}[\gamma] - \bar{\pi}^Y_{Passive}[\gamma]$$

$$= \frac{1+3\gamma+3\gamma^2}{6(1+\gamma)} - \left(\frac{1}{6} + \frac{\gamma}{3}\right)$$

$$= \frac{\gamma^2}{6(1+\gamma)}.$$

For reference only: Proposition 4

In first price auctions, the expected compound profit of the privileged bidder Y with value realization v_Y and bidding b_Y is

1) $\pi^{Y}_{Compound}\left[b_Y\right]=\int_0^{x[b_Y]} v_Y-(1-\gamma)b_Y dv_X+\int_{x[b_Y]}^1 \gamma b_X dv_X$

$$=x\left[b_Y\right]\left(v_Y-(1-\gamma)b_Y\right)+\gamma\left(\overline{b}-b_Y x[b_Y]-\int_{b_Y}^{\overline{b}} x[\beta]d\beta\right)$$, where $\overline{b}$ is the maximum bid.

The expected profit of the independent bidder X with value realization v_X and bidding b_X is

2) $\pi^X\left[b_X\right]=\int_0^{y[b_X]} v_X-b_X[v_X]dv_Y$

$$=y[b_X]\left(v_X-b_X\right).$$

Proof:

The privileged bidder wins the auction when his bid b_Y is bigger than the bid of the independent bidder. This is the case when $v_X<x[b_Y]$. The profit of the privileged bidder is then $\int_0^{x[b_Y]} v_Y-(1-\gamma)b_Y dv_X=x\left[b_Y\right]\left(v_Y-(1-\gamma)b_Y\right)$. The privileged bidder looes the auction, when $v_X>x[b_Y]$, his profit is then $\int_{x[b_Y]}^1 \gamma b_X[v_X]dv_X$. Because $v_X\equiv x[b_X]$ is a function of b_X, I can substitute for the variables in the integral. This results in $\int_{x[b_Y]}^1 \gamma b_X[v_X]dv_X=\gamma\int_{b_Y}^{\overline{b}} x'[\beta]\cdot\beta d\beta$. Integrating by parts then results in

$$\int_{x[b_Y]}^1 \gamma b_X[v_X]dv_X=\gamma\left(\overline{b}-b_Y x[b_Y]-\int_{b_Y}^{\overline{b}} x[\beta]d\beta\right).$$

Therefore, the compound profit of the privileged bidder Y is

$$\pi^{Y}_{Compound}\left[b_Y\right]=x\left[b_Y\right]\left(v_Y-(1-\gamma)b_Y\right)+\gamma\left(\overline{b}-b_Y x[b_Y]-\int_{b_Y}^{\overline{b}} x[\beta]d\beta\right).$$

The derivation is identical for the independent bidder X with γ set to zero.

Proposition 5

When in the first price auction allied bidder Y fully owns the auctioneer ($\gamma=1$) and faces n independent symmetric competing bidders, then the bidding function for each of the n independent bidders is

5') $b_X = \frac{n}{n+1} v_X$ for all v_X .

The bidding function for the allied bidder Y is

6') $b_Y = v_Y$.

Proof:
For the same reason as with one competing bidder, Y bids his own value when $\gamma = 1$, therefore $y[b] = b$. Substitute $x[b] = cb$ and $y[b] = b$ in equation 4) and we obtain

$$(x^{n-1}[b] \cdot y[b])' = \frac{x^{n-1}[b] \cdot y[b]}{x[b] - b}$$
$$\Leftrightarrow (c^{n-1} b^n)' = \frac{(cb)^{n-1} b}{(c-1)b}$$
$$\Leftrightarrow n(cb)^{n-1} = \frac{c^{n-1} b^n}{(c-1)b}$$
$$\Leftrightarrow c = 1 + \frac{1}{n} = \frac{n+1}{n}$$
$$\Leftrightarrow x[b] = \frac{n+1}{n} b \ .$$

Hence, the strategies $x[b] = \frac{n+1}{n} b$ solve the equations 3') and 4'). Taking inverses gives $b_X = \frac{n}{n+1} v_X$.

Proposition 6

Given a value of the ownership share, $\gamma : 0 < \gamma < 1$, the inverse bidding functions $x[b]$ and $y[b]$ and the maximum bid $\bar{b}$ should for all bids b fulfill:

1) $(y[b] - b) \cdot x'[b] = (1 - \gamma) x[b]$;

2) $(x[b] - b) \cdot y'[b] = y[b]$;

3) $x[\bar{b}] = y[\bar{b}] = 1$;

4) $\bar{b} = \frac{1}{2}\left(1 + \gamma \int_0^{\bar{b}} x[\beta] d\beta\right)$.

Proof:

Equation 1 and 2 are the first order conditions. Equation 3 states that a bidder only makes the maximum bid $\bar{b}$ when he has the highest possible value, which is one. This follows from the fact that it is a Nash-equilibrium to bid equal or lower than the highest bid. Equation 4 puts a restriction on the maximum bid that can be derived from the fact that a bidder with value 0 bids 0, $x[0] = y[0] = 0$, and the first order conditions 1) and 2).

Rewriting 1) and 2) gives

$$x'[b]\cdot(y[b]-b) = (1-\gamma)\cdot x[b] \Leftrightarrow$$

5) $$(x'[b]-1)\cdot(y[b]-b) = (1-\gamma)\cdot x[b]-y[b]+b\,,$$

$$y'[b]\cdot(x[b]-b) = y[b] \Leftrightarrow$$

6) $$(y'[b]-1)\cdot(x[b]-b) = y[b]-x[b]+b\,.$$

adding up 5) and 6) gives;

$$(x'[b]-1)\cdot(y[b]-b)+(y'[b]-1)\cdot(x[b]-b) = 2b-\gamma x[b] \Leftrightarrow$$

7) $$\frac{\partial}{\partial b}(x[b]-b)\cdot(y[b]-ab) = 2b-\gamma x[b].$$

Integrating equation 7) over 0 to the maximum bid $\bar{b}$ gives

$$(1-\bar{b})\cdot(1-\bar{b}) = \bar{b}^2-\gamma\int_0^{\bar{b}} x[b] \Leftrightarrow$$

$$1+\bar{b}^2-2\bar{b} = \bar{b}^2-\gamma\int_0^{\bar{b}} x[b] \Leftrightarrow$$

4) $$\bar{b} = \frac{1}{2}\left(1+\gamma\int_0^{\bar{b}} x[b]\right).$$

Proposition 7

All effects with * are ex-ante expected measure (before bidding and before concrete values have been realized).

a. The probability of winning* for the privileged bidder Y is equal to

$$p^{Y\ wins}[\gamma = 1] = \frac{3}{4}\,.$$

Proof:

Using proposition 1 and 3, it follows that the privileged bidder Y with a realized value of v_Y wins with probability

$$p^{Y\,wins}[v_Y] = x \circ b_Y[v_Y] = 2 \cdot v_Y \qquad \text{when } v_Y \le \frac{1}{2}$$

$$p^{Y\,wins}[v_Y] = 1 \qquad \text{when } v_Y > \frac{1}{2}.$$

The expected proportion of auctions that is won by the privileged bidder Y is therefore

$$p^{Y\,wins}[1] = \int_0^1 p^{Y\,wins}[v_Y;1]dv_Y$$

$$= \int_0^{\frac{1}{2}} 2v_Y\,dv_Y + \int_{\frac{1}{2}}^1 1dv_Y$$

$$= \frac{3}{4}.$$

b. The generation profit* of Y is equal to $\pi^Y_{Generator}\left[\gamma = 1\right] = 0$.

Proof:

As Y bids his own value, his profit is equal to zero.

c. The revenue* of the auctioneer is equal to $m\left[\gamma = 1\right] = \frac{13}{24}$.

Proof:

$$m\left[\gamma\right] = m^Y\left[\gamma\right] + m^X\left[\gamma\right]$$

$$m\left[\gamma\right] = \int_0^{\frac{1}{2}} P^{Y\,WINS}\left(b_Y\left[v_Y\right]\right)dv_Y + \int_{\frac{1}{2}}^1 P^{Y\,WINS}\cdot\left(b_Y\left[v_Y\right]\right)dv_Y + \left(\int_0^1 P^{X\,WINS}\left(b_X\left[v_X\right]\right)dv_X\right)$$

$$= \int_0^{\frac{1}{2}} 2v_Y\left(v_Y\right)dv_Y + \int_{\frac{1}{2}}^1 1\cdot\left(v_Y\right)dv_Y + \left(\int_0^1 \frac{v_x}{2}\left(\frac{v_x}{2}\right)dv_x\right)$$

$$= \left[\frac{2v_Y^3}{3}\right]_0^{\frac{1}{2}} + \left[\frac{1}{2}v_Y^2\right]_{\frac{1}{2}}^1 + \left(\left[\frac{v_x^3}{12}\right]_0^1\right)$$

$$= \frac{1}{12} + \frac{1}{2} - \frac{1}{8} + \left(\frac{1}{12}\right)$$

$$= \frac{13}{24}.$$

d. The compound profit* of Y is equal to $\pi^{Y}_{Compound}\left[\gamma=1\right]=\frac{13}{24}$.

Proof:

$$\pi^{Y}_{Compound}\left[\gamma\right]=\pi^{Y}_{Generator}\left[\gamma\right]+\gamma m\left[\gamma\right].$$

Therefore, $\pi^{Y}_{Compound}\left[\gamma=1\right]=\frac{13}{24}$.

e. The profit* of the independent bidder X is equal to $\pi^{X}\left[\gamma=1\right]=\frac{1}{12}$.

Proof:

$$\pi^{X}\left[\left[\gamma=1\right]\right]=\int_0^1 P^{X\,WINS}\left(v_x-b_x\left[v_x\right]\right)dv_x$$

$$=m^{X}\left[\gamma=1\right]$$

$$=\frac{1}{12}.$$

f. The welfare* is equal to $W\left[\gamma=1\right]=\frac{5}{8}$.

Proof:

$$W\left[\gamma\right]=m\left[\gamma\right]+\pi^{X}\left[\gamma\right]+\pi^{Y}_{Generator}\left[\gamma\right]$$

$$W\left[\gamma=1\right]=\frac{13}{24}+\frac{1}{12}=\frac{5}{8}.$$

g. The strategic profit* of Y is equal to $\pi^{Y}_{Strategic}\left[\gamma=1\right]=\tfrac{1}{24}$.

Proof:

Using $\overline{\pi}^{Y}_{Passive}\left[\gamma\right]=\pi^{Y}\left[0\right]+\gamma m\left[0\right]=\frac{1}{6}+\frac{1}{3}=\frac{1}{2}$, gives

$$\pi^{Y}_{Strategic}\left[\gamma\right]=\pi^{Y}_{Compound}\left[\gamma\right]-\overline{\pi}^{Y}_{Passive}\left[\gamma\right]$$

$$\pi^{Y}_{Strategic}\left[\gamma=1\right]=\tfrac{13}{24}-\tfrac{1}{2}=\tfrac{1}{24}.$$

6. Parameter overview

γ	$\gamma \in [0,1]$ is the ownership share that the privileged bidder holds in the auctioneer. The privileged bidder therefore receives the portion γ of the revenue of the auctioneer.
b	$b \in [0,\bar{b}] \subseteq [0,1]$ is the officially stated bid offered by a bidder. $\bar{b} \in [0,1]$ is the maximum bid in the auction.
$b_Y[v]$	The optimal bid of the privileged bidder Y given his realized value $v \in [0,1]$. This strategy $b_Y[\cdot]$ has the inverse $y[\cdot]$ (such that $y[b_Y[v]] = v$).
$b_X[v]$	$b_X[v]$ is the optimal bid of the independent bidder X given her realized value $v \in [0,1]$. This strategy $b_X[v]$ has the inverse $x[\cdot]$ (such that $x[b_X[v]] = v$).
$p^{Y\,WINS}[\gamma]$	The ex-ante expected probability that the privileged bidder Y wins the auction when using his optimal strategy $b_Y[\cdot]$, given his ownership share γ .
$m^Y[\gamma]$	The ex-ante expected payment of the privileged bidder Y when the ownership share is γ .
$m[\gamma]$	$m[\gamma] = m^Y[\gamma] + m^X[\gamma]$ is the ex-ante expected revenue of the auctioneer when the ownership share is γ .
v	$v \in [0,1]$ is the value of the good in the auction. It is a random variable uniformly distributed on $[0,1]$.
$W[\gamma]$	The ex-ante expected welfare. It is the value of the good in use by the bidder that won the auction.
$x[\cdot]$	The inverse of strategy $b_X[v]$ (such that $x[b_X[v]] = v$).
$y[\cdot]$	The inverse of strategy $b_Y[\cdot]$ (such that $y[b_Y[v]] = v$).
$\pi_{auction}[\gamma]$	The ex-ante expected profit of the auctioneer.

$\pi^{Y}_{Compound}[\gamma]$ The expected compound profit of the privileged bidder Y when using his (optimal) strategy $b_Y[\cdot]$, given his realized value v_Y and the ownership share γ.

$\overline{\pi}^{Y}_{Passive}[\gamma]$ $\overline{\pi}^{Y}_{Passive}[\gamma] = \pi^{Y}_{Compound}[0] + \gamma m[0]$ is the ex-ante expected passive compound profit of the privileged bidder. It is the compound profit when the privileged bidder has an ownership share of γ, but bids as if the ownership share is zero (she maximizes his own private profit).

$\pi^{Y}_{Strategic}[\gamma]$ $\pi^{Y}_{Strategic}[\gamma] = \pi^{Y}_{Compound}[\gamma] - \overline{\pi}^{Y}_{Passive}[\gamma]$ is the ex-ante expected strategic profit. It is the extra profit that can be made when the privileged bidder Y maximizes the compound profit instead of his private profit.

7. Literature

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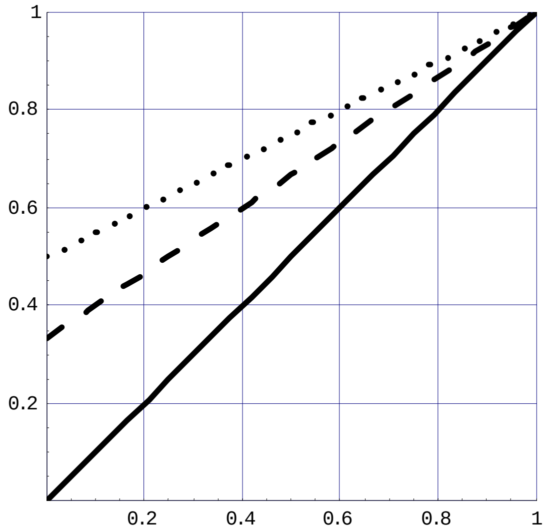
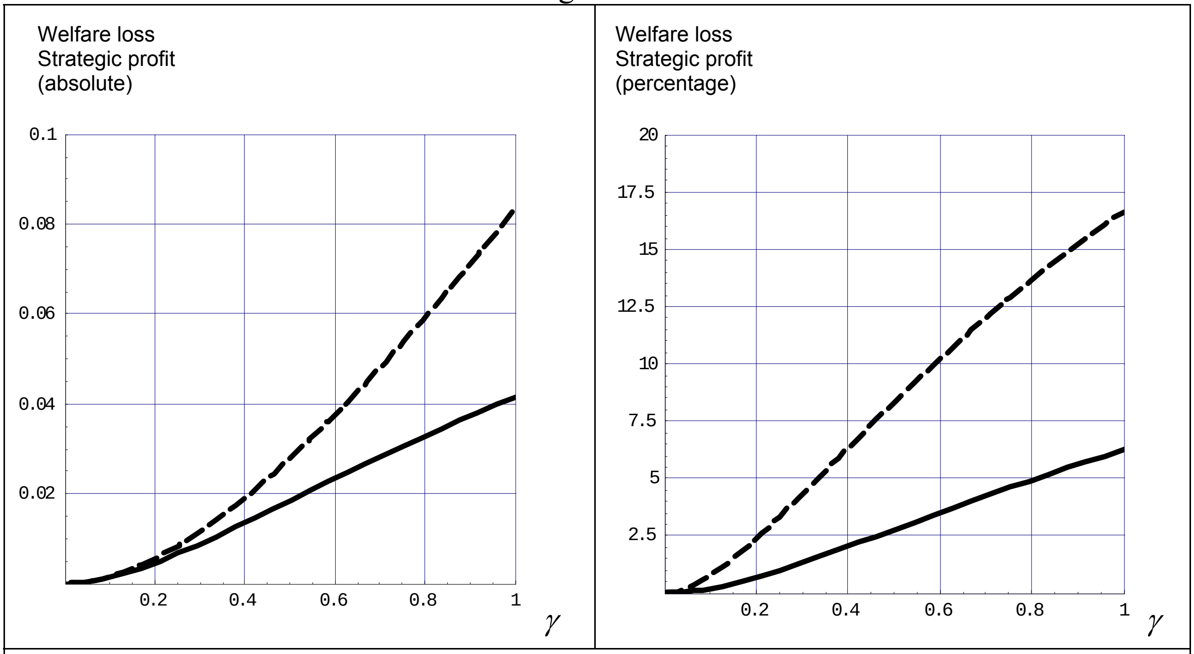
Figure 1



… bidding function of Y when $\gamma = 1$
--- bidding function of Y when $\gamma = 0.5$
— bidding function of Y when $\gamma = 0$

Figure 2

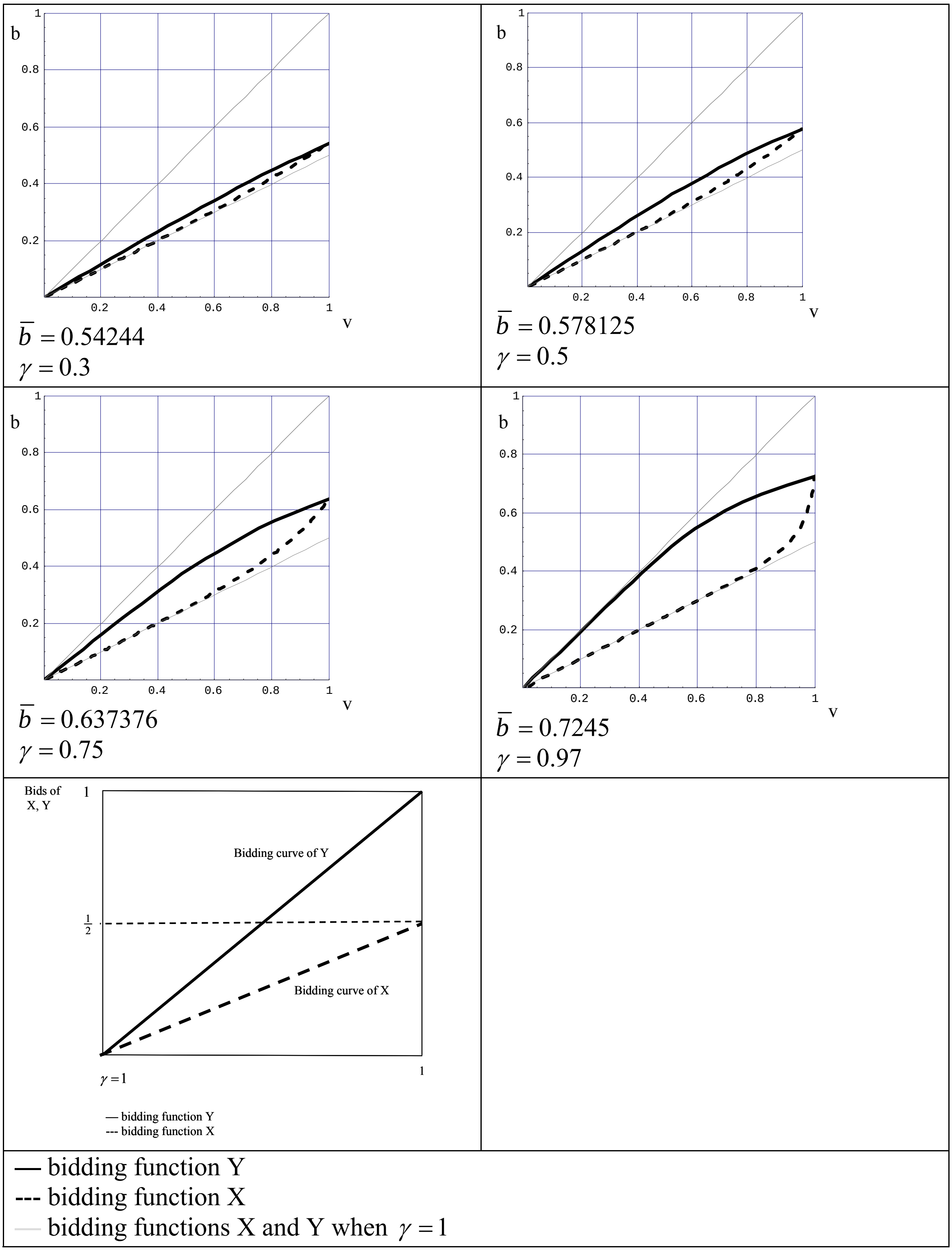


--- Strategic profit of Y[28]
— Welfare loss[29]

[28] The welfare loss percentage is calculated as $\frac{W[0]-W[\gamma]}{W[0]}$.

[29] The strategic profit percentage is calculated as $\frac{\pi^{Y}_{Strategic}}{\bar{\pi}^{Y}_{Passive}}$.

Figure 3: the bidding functions for X and Y.



Individual researchers, as well as the on-line and printed versions of the CERGE-EI Working Papers (including their dissemination) were supported from the following institutional grants:

- Center of Advanced Political Economy Research [Centrum pro pokročilá politicko-ekonomická studia], No. LC542, (2005-2009),
- Economic Aspects of EU and EMU Entry [Ekonomické aspekty vstupu do Evropské unie a Evropské měnové unie], No. AVOZ70850503, (2005-2010);
- Economic Impact of European Integration on the Czech Republic [Ekonomické dopady evropské integrace na ČR], No. MSM0021620846, (2005-2011);

Specific research support and/or other grants the researchers/publications benefited from are acknowledged at the beginning of the Paper.

Published by
Charles University in Prague, Center for Economic Research and Graduate Education (CERGE)
and
Economics Institute (EI), Academy of Sciences of the Czech Republic
CERGE-EI, Politických vězňů 7, 111 21 Prague 1, tel.: +420 224 005 153, Czech Republic.
Printed by CERGE-EI, Prague
Subscription: CERGE-EI homepage: http://www.cerge-ei.cz

Editors: Directors of CERGE and EI
Managing editors: Deputy Directors for Research of CERGE and EI

ISSN 1211-3298
ISBN 80-7343-109-2 (Univerzita Karlova. Centrum pro ekonomický výzkum a doktorské studium)
ISBN 80-7344-098-9 (Akademie věd České republiky. Národohospodářský ústav)

CERGE-EI
P.O.BOX 882
Politických vězňů 7
111 21 Praha 1
Czech Republic
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